A.P. Physics B – Circular Motion Problems

1. (1974, 1) Diagram, below left

A pendulum consisting of a small heavy ball of mass $m$ at the end of a string of length $l$ is released from a horizontal position. When the ball is at point $P$, the string forms an angle of 30° with the horizontal as shown above.

(a) In the space below, draw a force diagram showing all of the forces acting on the ball at $P$. Identify each force clearly.
(b) Determine the speed of the ball at $P$.
(c) Determine the tension in the string when the ball is at $P$.

2. (1975, 7) Diagram, above right

A pendulum consists of a small object of mass $m$ fastened to the end of an inextensible cord of length $L$. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

(a) In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

(b) Determine the tension in the cord before the string is burned.
(c) Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.
(d) The motion of the pendulum after the string is burned is periodic. Is it also simple harmonic? Why, or why not?

3. (1977, 2)

A box of mass $M$, held in place by friction, rides on the flatbed of a truck which is traveling with constant speed $v$. The truck is on an unbanked circular roadway having radius of curvature $R$.

(a) On the diagram provided above, indicate and clearly label all the force vectors acting on the box.
(b) Find what condition must be satisfied by the coefficient of static friction $\mu$ between the box and the truck bed. Express your answer in terms of $v$, $R$, and $g$.

If the roadway is properly banked, the box will still remain in place on the truck for the same speed $v$ even when the truck bed is frictionless.

(c) On the diagram below, indicate and clearly label the two forces acting on the box under these conditions.

(d) Which, if either, of the two forces acting on the box is greater in magnitude? Explain.
4. (1978, 1) Diagram, below left

A 0.5-kilogram object rotates freely in a vertical circle at the end of a string of length 2 meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is 20 newtons. Assume \( g = 10 \text{ meters per second squared.} \)

(a) On the following diagram of the object, draw and clearly label all significant forces on the object when it is at the point P. 

(b) Calculate the speed of the object at point P.
(c) Calculate the increase in kinetic energy of the object as it moves from point P to point Q.
(d) Calculate the tension in the string as the object passes through point Q.

5. (1980, 1) Diagram, above right

A ball of weight 5 newtons is suspended by two strings as shown above.

(a) In the space below, draw and clearly label all the forces that act on the ball.
(b) Determine the magnitude of each of the forces indicated in part (a). (Note: \( \sin 37^\circ = 0.6; \cos 37^\circ = 0.8 \))

Suppose that the ball swings as a pendulum perpendicular to the plane of the page, achieving a maximum speed of 0.6 meter per second during its motion.

(c) Determine the magnitude and direction of the net force on the ball as it swings through the lowest point in its motion. Use \( g = 10 \text{ m/s}^2 \).

6. (1982, 3) Diagram, above right

A child of mass \( M \) holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length \( R \) and negligible mass. The initial angle of the rope with the vertical is \( \theta_0 \), as shown in the drawing above.

(a) Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of \( g \), \( R \), and \( \cos \theta_0 \).
(b) The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of \( \cos \theta_0 \).
7. (1984, 1) Diagram below left

A ball of mass $M$ attached to a string of length $L$ moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of $M$, $L$, and $g$.

(a) Determine the magnitude and direction of the net force on the ball when it is at the top.
(b) Determine the speed $v_0$ of the ball at the top.
(c) Determine the time it takes the ball to reach the ground.
(d) Determine the horizontal distance the ball travels before hitting the ground.

8. (1989, 1) Diagram above right

An object of mass $M$ on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed $v_0$ and the circular path has radius $R$ and is a height $h$ above the ground. Neglect air friction.

(a) Determine the following, expressing all answers in terms of $h$, $v_0$, and $g$.
   i. The time required for the object to hit the ground after the string breaks
   ii. The horizontal distance the object travels from the time the string breaks until it hits the ground
   iii. The speed of the object just before it hits the ground
(b) On the figure below, draw and label all the forces acting on the object when it is in the position shown in
(c) Determine the tension in the string just before the string breaks. Express your answer in terms of $M$, $R$, $v_0$, and $g$.

9. (1992, 1)

A 0.10-kilogram solid rubber ball is attached to the end of an 0.80-meter length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point $P$, the lowest point of the circle, is 0.20 meter above the floor. The speed of the ball at the top of the circle is 6.0 meters per second, and the total energy of the ball is kept constant.

(a) Determine the total energy of the ball, using the floor as the zero point for gravitational potential energy.
(b) Determine the speed of the ball at point $P$, the lowest point of the circle.
(c) Determine the tension in the thread at
   i. the top of the circle;
   ii. the bottom of the circle.
9. continued

The ball only reaches the top of the circle once before the thread breaks when the ball is at the lowest point of the circle.

(d) Determine the horizontal distance that the ball travels before hitting the floor.

10. (1995, 3)

Part of the track of an amusement park roller coaster is shaped as shown above. A safety bar is oriented lengthwise along the top of each car. In one roller coaster car, a small 0.10-kilogram ball is suspended from this bar by a short length of light, inextensible string.

(a) Initially, the car is at rest at point A.

i. On the diagram to the right, draw and label all the forces acting on the 0.10-kilogram ball.

ii. Calculate the tension in the string.

The car is then accelerated horizontally, goes up a 30° incline, goes down a 30° incline, and then goes around a vertical circular loop of radius 25 meters. For each of the four situations described in parts (b) to (e), do all three of the following. Assume that the ball has stopped swinging back and forth.

- Determine the horizontal component $T_h$ of the tension in the string in newtons and record your answer in the space provided.
- Determine the vertical component $T_v$ of the tension in the string in newtons and record your answer in the space provided.
- Show on the adjacent diagram the approximate direction of the string with respect to the vertical. The dashed line shows the vertical in each situation.

(b) The car is at point $B$ moving horizontally to the right with an acceleration of 5.0 m/s².

(c) The car is at point $C$ and is being pulled up the 30° incline with a constant speed of 30 m/s.

(d) The car is at point $D$ moving down the 30° incline with an acceleration of 5.0 m/s².

(e) The car is at point $E$ moving upside down with an instantaneous speed of 25 m/s and no tangential acceleration at the top of the vertical loop of radius 25 meters.
Answers to AP Circular Motion Problems:

1. b. $\sqrt{gl}$
   
2. b. 2mg

3. b. $\mu \geq \frac{v^2}{Rg}$

4. b. 10 m/s
c. 20 J
d. 50 N

5. b. $T_1 = 4$ N, $T_2 = 3$ N
c. 1.5 N

6. a. $v = \sqrt{2gR(1 - \cos \theta_0)}$
b. $\cos \theta = \frac{3}{4}$

7. a. 3Mg downward
   
   b. $v_0 = \sqrt{3Lg}$
   
   c. $t = 2\sqrt{\frac{L}{g}}$
   
   d. $2\sqrt{3L}$

8. a. i. $t = \sqrt{\frac{2h}{g}}$
   
   ii. $x = v_0 \sqrt{\frac{2h}{g}}$
   
   iii. $v = \sqrt{v_0^2 + 2gh}$
   
   b. $F_T = M\sqrt{g^2 + \frac{v_0^4}{R^2}}$

9. a. $E_{total} = 3.56$ J
   
   b. $v_f = 8.2$ m/s
   
   c. $F_T = 3.5$ N
   
   d. $\Delta x = 1.6$ m

10. a. ii. $F_T = 1$ N
    
    b. $F_{Th} = 0.5$ N, $F_{Tv} = 1$ N
    
    c. $F_{Th} = 0, F_{Tv} = 1$ N
    
    d. $F_{Th} = 0.43$ N, $F_{Tv} = 0.75$ N
    
    e. $F_{Th} = 0, F_{Tv} = 1.5$ N